Rabobank

Eurobanking 2011 Vienna
Credit risk; Name concentration in Basel II

On the move

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Basel II regulatory capital

1. Pillar I

The Regulatory capital model for credit risk assumes (amongst others):

- 1 factor model
- Infinitely granular portfolio

Results is an analytical formula for the Value at Risk for credit risk. The VaR is the 99.9%th quantile of the loss distribution.

Basel II regulatory capital

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1. Pillar II

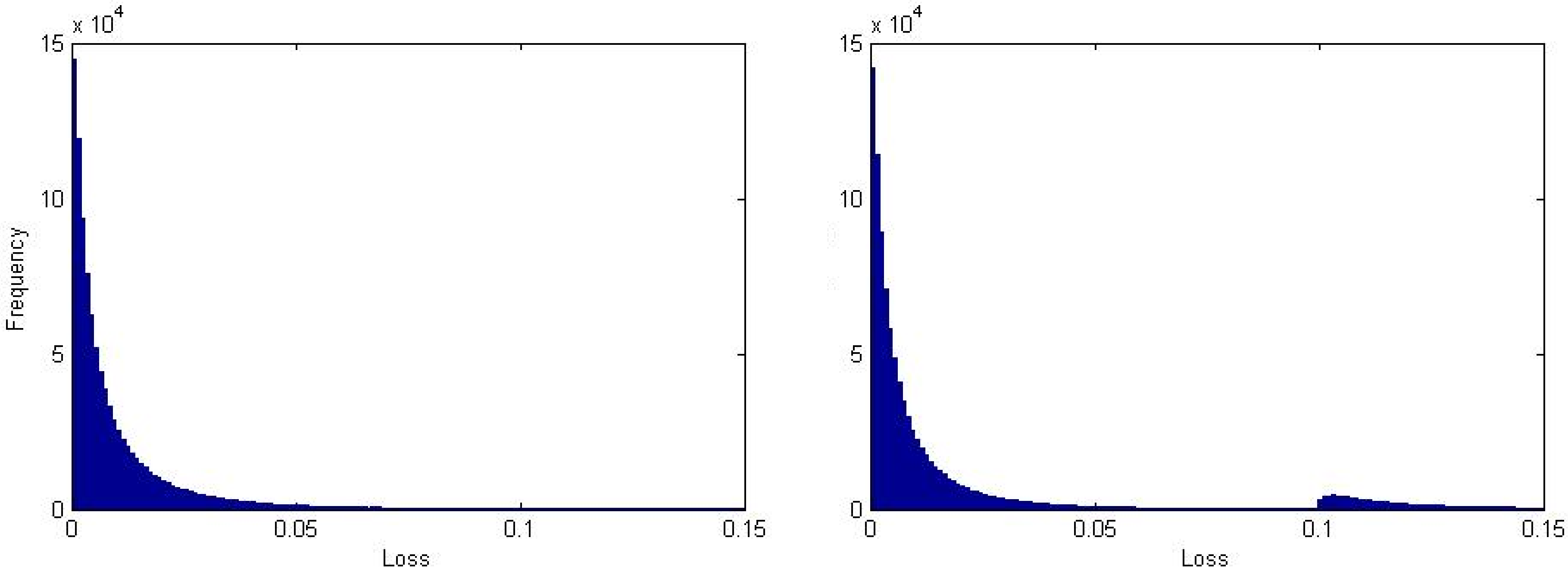
In article 773 of the Basel II accord 3 sources of concentration are mentioned:

- Name concentration
- Region/industry concentration
  - Correlation of defaults
- Indirect exposure concentration: collateral
  - Correlation of default and losses in case of default

Some general introduction in: Basel document Working Paper No. 15 Studies on credit risk concentration

Examples name concentration

Loss distribution function of a fully granular portfolio (left) and of a fully granular portfolio with a single large exposure (right).

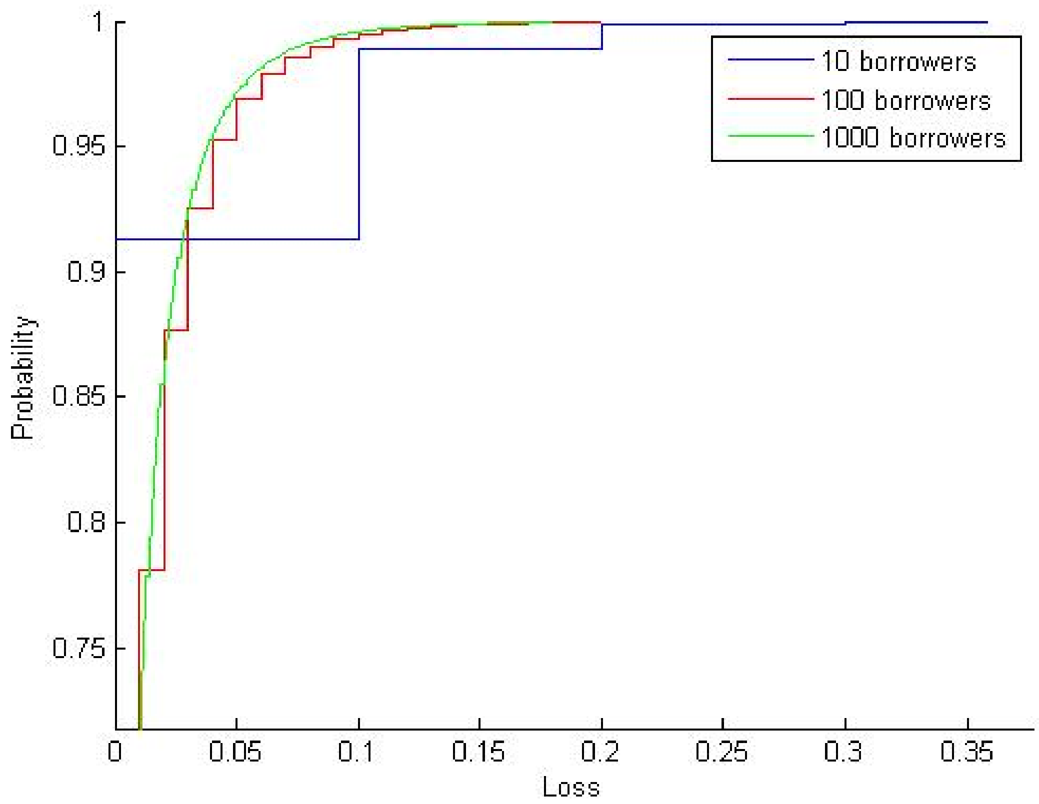


Hump is caused by name concentration, when borrower of large exposure defaults, loss is large.

Examples name concentration

Homogenous portfolio (in PD, LGD and correlation) evenly divided over changing number of borrowers (10,100 and 1000).

Graph shows cumulative loss distribution of the three portfolios.



Pd	1%
Asset correlation	20%
LGD	100%
Port exposure	1

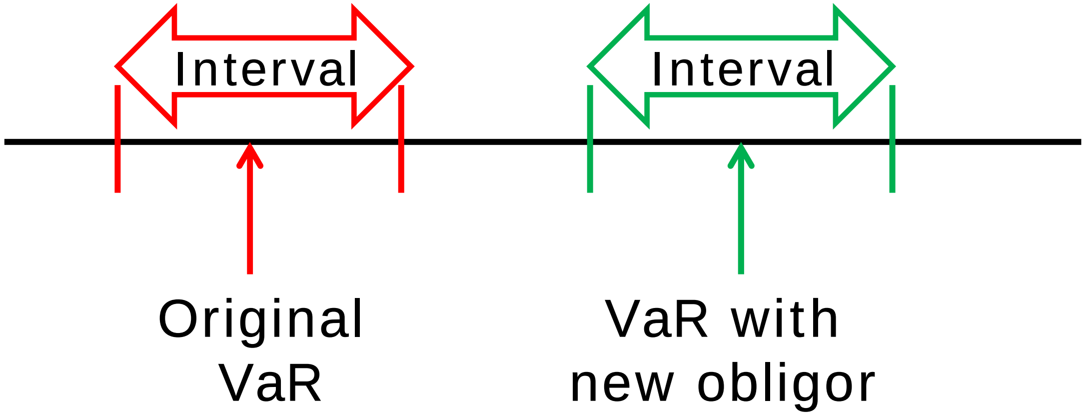
Nr obligors	VaR (99.9%)
1	100%
5	40%
10	30%
50	18%
100	16%
500	15%
1000	15%

Need for analytical solution

Value at Risk for a portfolio can be determined with a Monte Carlo simulation.

Monte Carlo results include simulation noise.

When a new obligor is added to the portfolio the noise in incremental VaR can become very large.



		error interval
MC A	1,000,000,000.00	1%
MC A + new borrower	1,010,000,000.00	1%
Incremental	10,000,000.00	142%

Description analytical solution

The IRB capital model is based on the following elements

- Merton model for defaults
  - A counterparty defaults when its asset return falls below a certain threshold at the horizon (1 year)
  - The assets follow a Geometric Brownian Motion
- One factor model. The asset returns of different counterparties are influenced by one global factor.
- Perfect granular portfolio.

Detailed description can be found in publication in RISK magazine of December 2010.

Description analytical solution

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IRB model in equations:

Asset return in normally distributed and driven by 1 global factor

$$X_i = \sqrt{\rho_i} Y + \sqrt{1-\rho_i}\, \varepsilon_i$$

Default occurs when asset return falls below a threshold

$$X_i < N^{-1}\left(PD_i\right)$$

Conditional default probability given value of Y

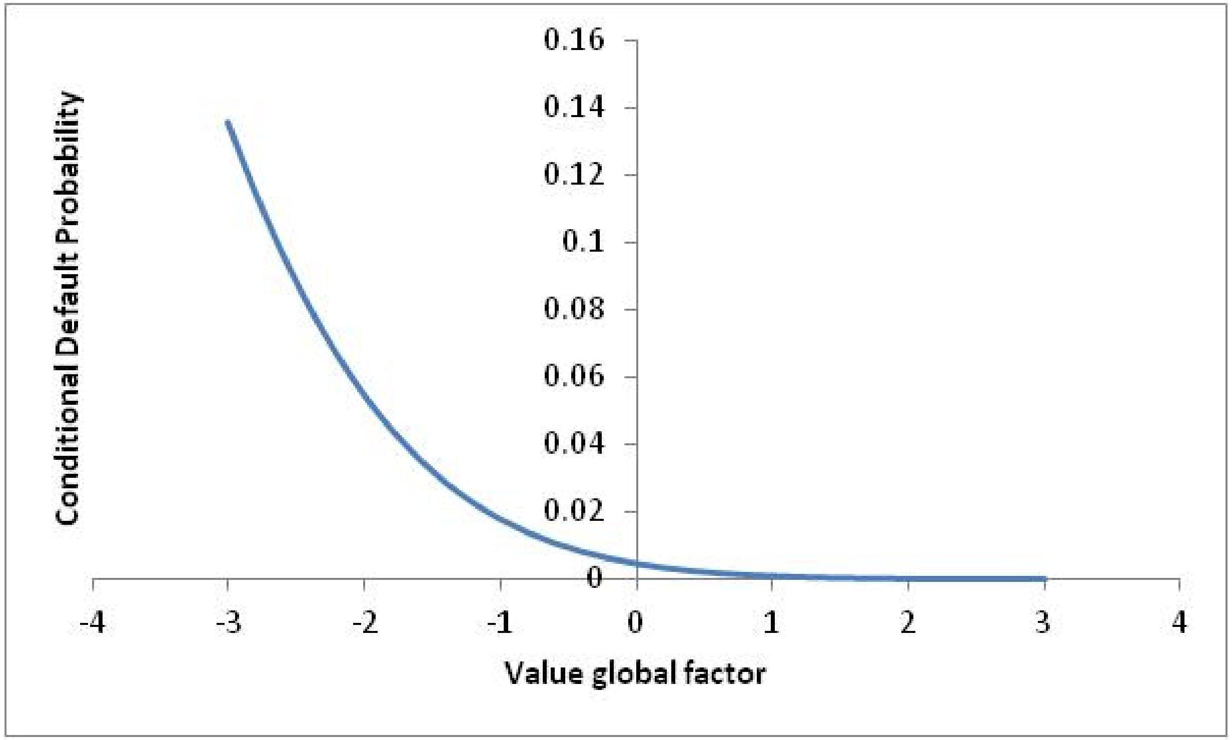
$$CPD_i(Y = y) = N\left(\frac{N^{-1}(PD_i) - \sqrt{\rho_i}\, y}{\sqrt{1-\rho_i}}\right)$$

In a perfect granular portfolio, the portfolio loss is only driven by the global factor:

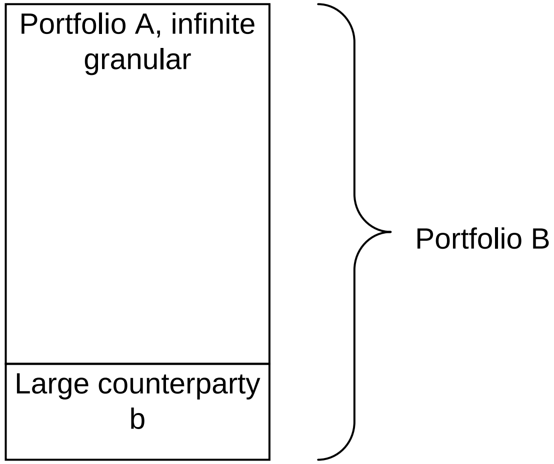
$$\left(L_{\text{portfolio}} \mid Y = y\right) = \sum_i CPD_i * LGD_i * EAD_i$$

Description analytical solution

So in a perfect granular portfolio the loss is only driven by the value of Y.



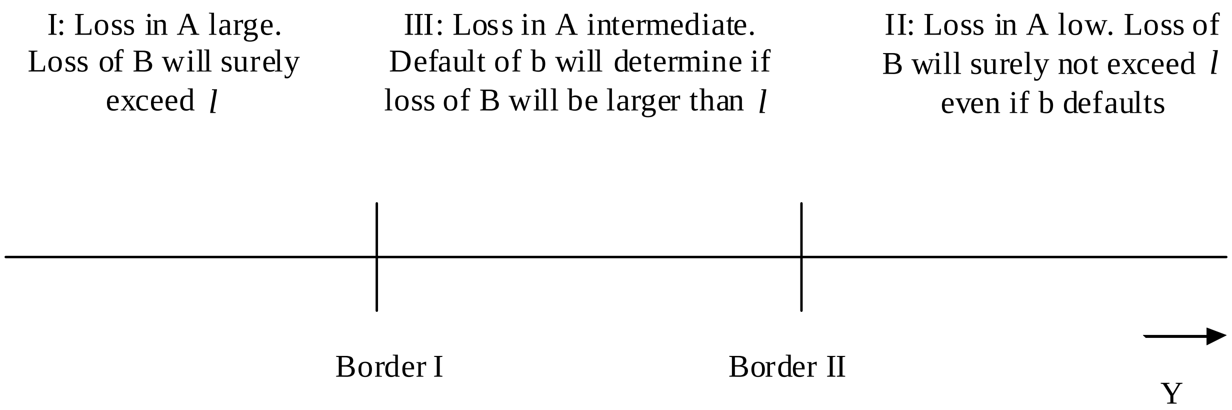
Description analytical solution



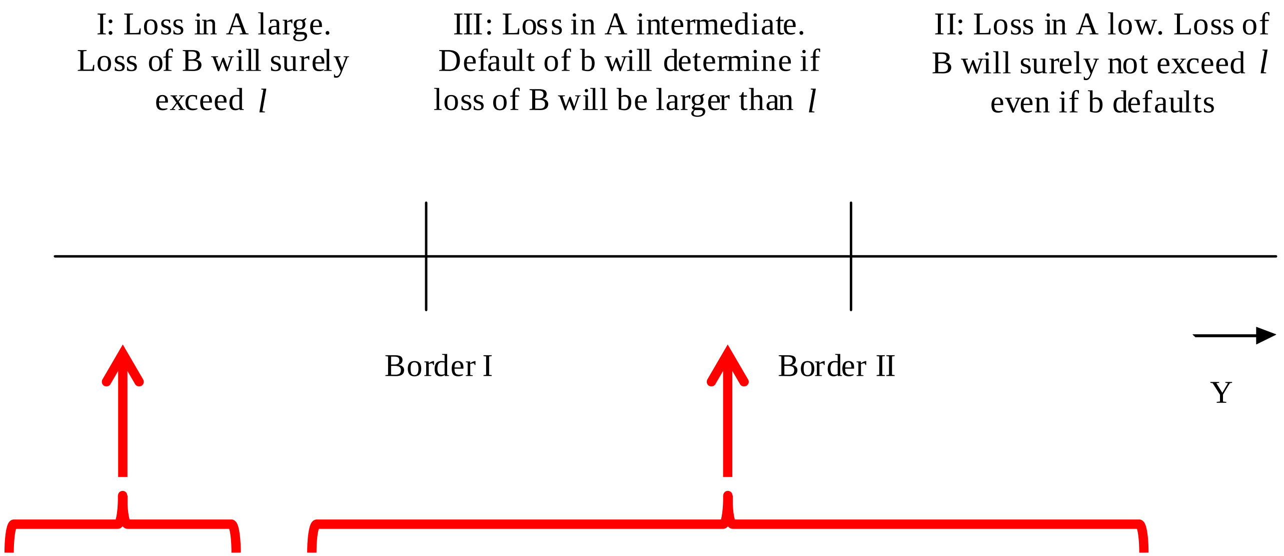
Now take a portfolio B consisting of:

- Portfolio A, perfectly granular
- One large counterparty b.

What is probability that the loss of portfolio B exceeds l?



Description analytical solution



$$P(L > l) = \int_{-\infty}^{y_1(l)} \varphi(y)dy - \int_{y_1(l)}^{y_2(l)} CPD_b(y)\varphi(y)dy$$

So the cumulative loss distribution of portfolio B is given by the cumulativee loss distribution of A (perfectly granular) minus a correction probability

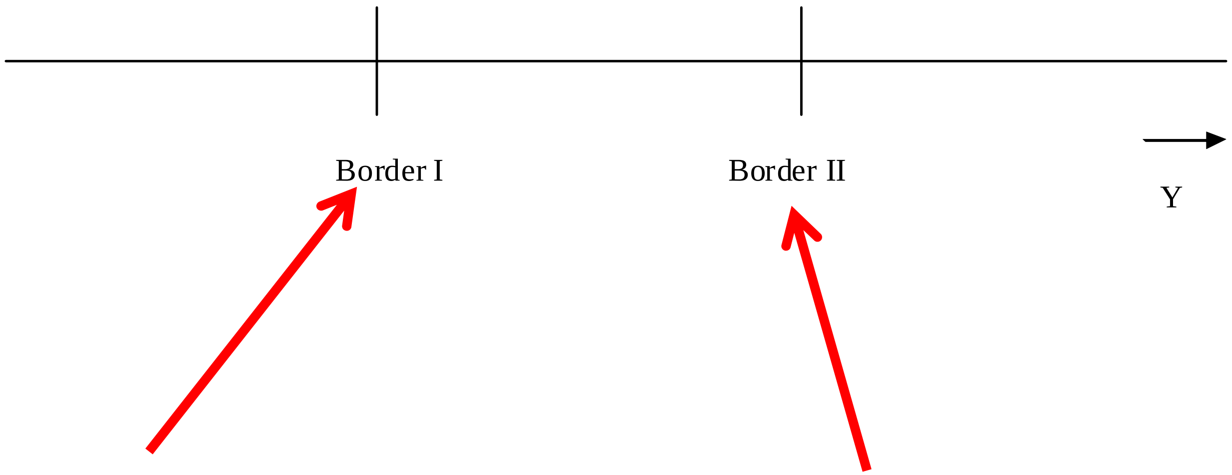
$$F_B(l) = P[L_B \leq l] = P[L_A \leq l] - \Delta P(l)$$

Description analytical solution

I: Loss in A large. Loss of B will surely exceed l

III: Loss in A intermediate. Default of b will determine if loss of B will be larger than l

II: Loss in A low. Loss of B will surely not exceed l even if b defaults



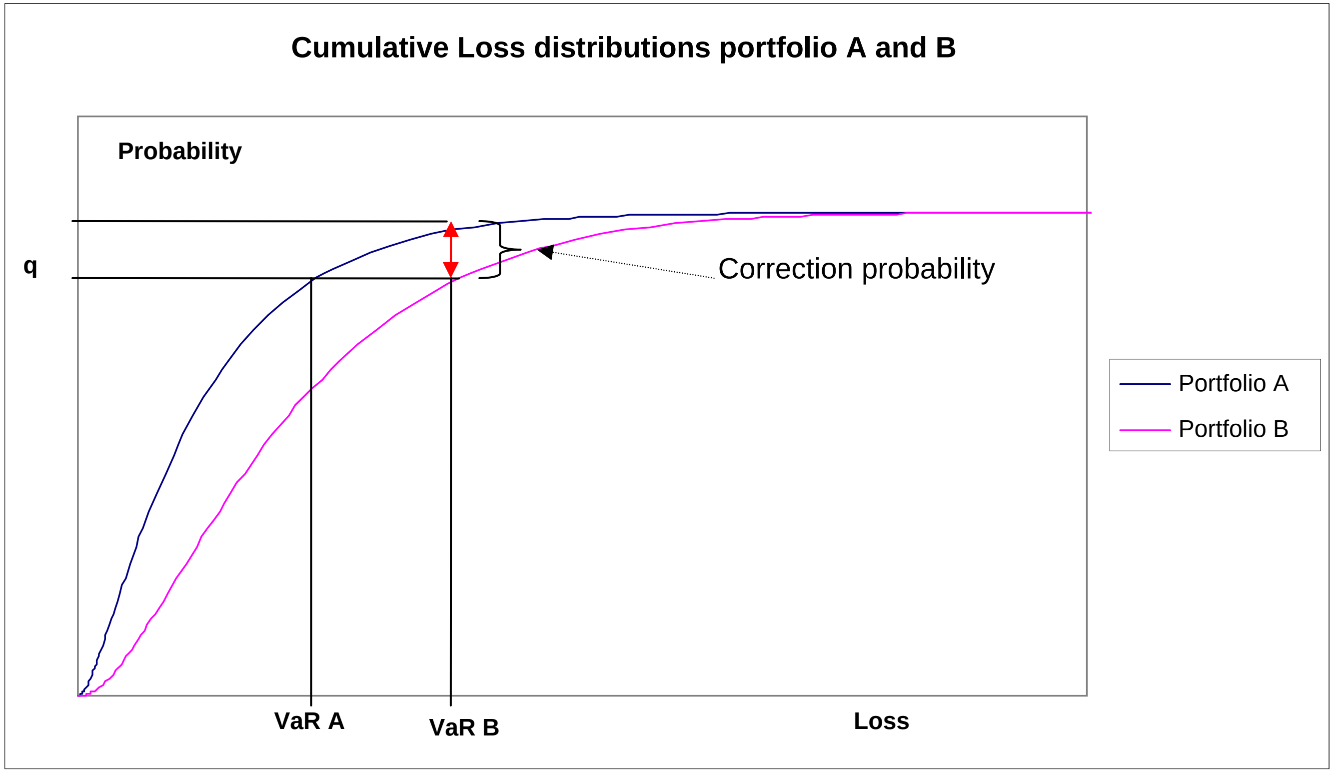
$$l = \sum_i CPD_i(y_1) * LGD_i * EAD_i$$

$$l - LGD_B EAD_B = \sum_i CPD_i(y_2) * LGD_i * EAD_i$$

When portfolio A is homogenous in PD and ρ then y1 and y2 can be found analytically otherwise use a rootfinding algorithm.

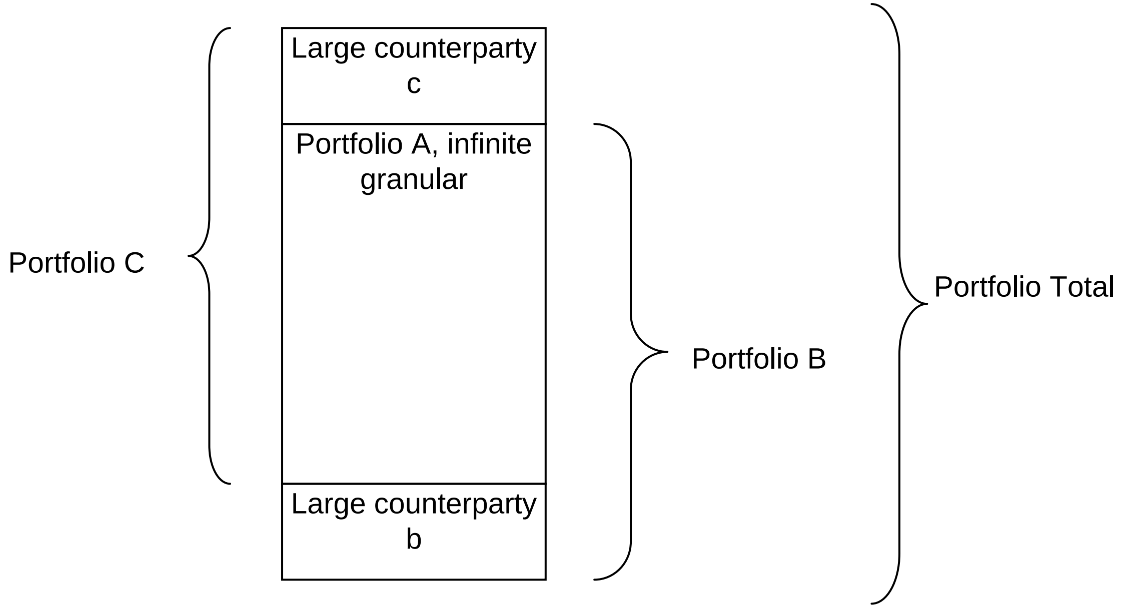
Description analytical solution

$$F_B(l) = P[L_B \leq l] = P[L_A \leq l] - \Delta P(l)$$



Description analytical solution

Extension to two large counterparties



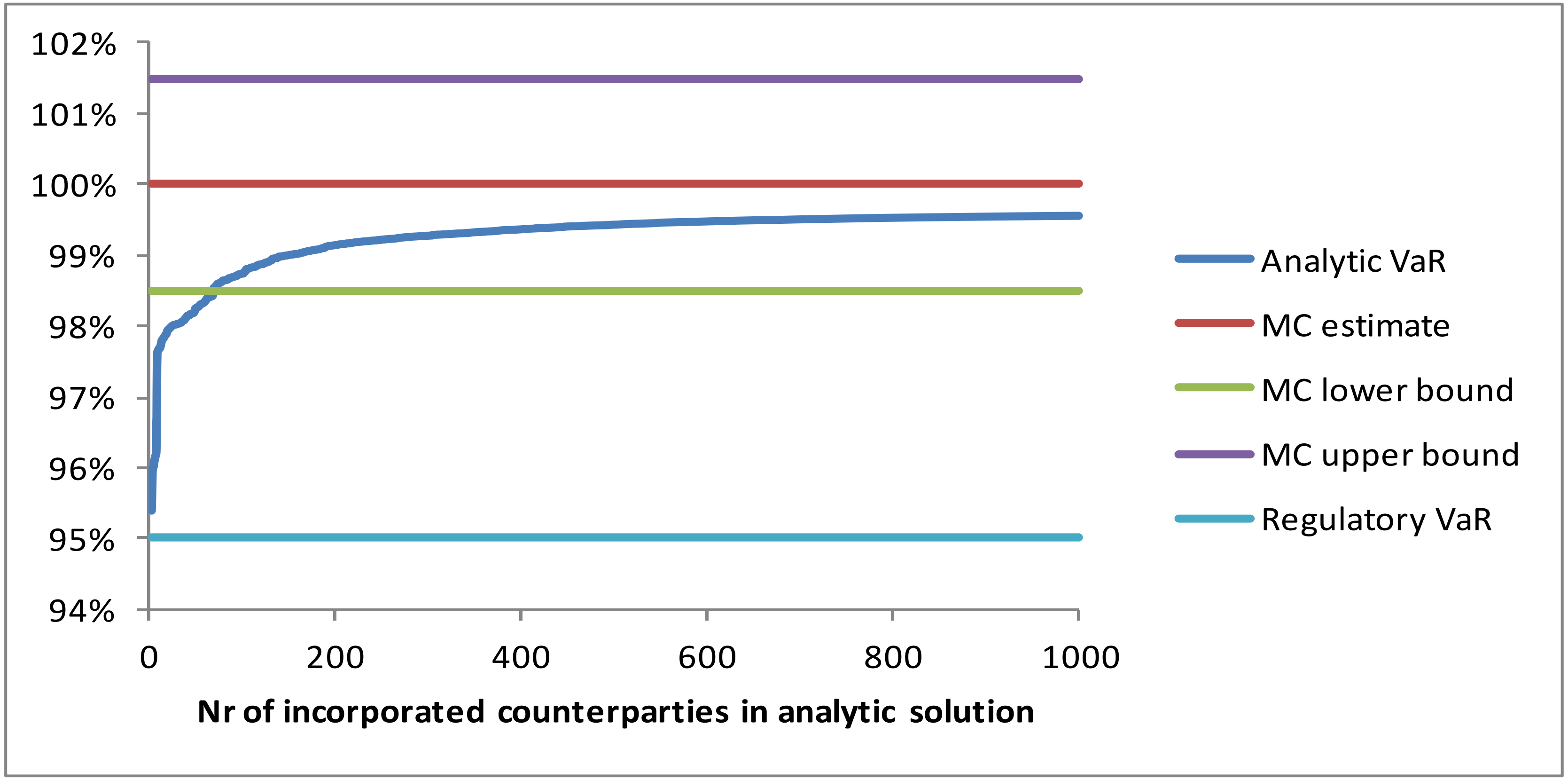
$$\Delta VaR_b = VaR_{Total,b} - VaR_{C,Vasicek}$$

$$VaR_{Total} \approx VaR_{A,Vasicek} + \Delta VaR_b + \Delta VaR_c$$

Description analytical solution

Extension to multiple counterparties:

$$VaR_{Total}(n) = VaR_{A,Vasicek} + \sum_{i=1}^{n} \Delta VaR_i$$



Performance approximation

Comparison analytical approximation with MC results.

Portfolio consist of a perfect granular part with EAD of 10.000 Euro. 10 ‘Large’ counterparties are added to the portfolio with increasing EAD.

PD=1% and LGD=100%.

EAD large counterparty	Ratio exposure of large counterparties to total portfolio	VaR analytic	VaR MC	Error margin MC	Deviation MC and analytic
20	1.96%	767.99	771.00	1.39%	-0.39%
40	3.85%	784.32	786.00	1.39%	-0.21%
100	9.09%	838.53	840.00	1.39%	-0.18%
200	16.67%	946.24	945.00	1.39%	0.13%
400	28.57%	1225.70	1200.00	1.39%	2.10%

Questions

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