Inflation Hedging

Karim Mzoughi, Ernesto Palidda
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Groupe de Recherche Operationnelle

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Motivation

- French banks offer saving accounts that are directly or indirectly inflation indexed.
- The most popular (75% of the French population) inflation-linked product is Livret A which provides a performance above inflation.
- These products represent huge positions (€320-billion-euros in 2009) in the balance sheet of French retail banks. Therefore ALM practitioners have to measure and hedge inflation risk with particular attention.

Motivation : cont'd

The Inflation market proposes two kinds of products to hedge inflation risk:

- Inflation-linked bonds (OATi, OAT€i, Corporate Inflation Bonds)
- Exotic inflation derivatives (zero-coupon inflation, year-on-year swap, calendar spread, inflation cap/floor...)

However there is a liquidity risk. Indeed the disequilibrium between supply and demand (structural poor supply and strong hedging demand) leads to high bid-ask spreads.

In this context of market distortion, we cannot perfectly hedge the inflation risk.

Inflation

- The financial market offers now various securities indexed to inflation. But inflation, as an underlying, is a specific asset : indeed the inflation is not a rate quoted in the market provided by supply and demand equilibrium.
- Inflation is measured as the relative variation of the Consumer Price Index (CPI) which is an average of the prices of a standard set of goods. It is published monthly by the statistics office (INSEE).

Livret A : a passbook saving account

Livret A is a passbook saving account created in 1818 by King Louis XVIII. It is exempt from income tax and have an interest rate that is set every six months by the government since 2004.

We display afterward the evolution of the formula modified by the French government. Nevertheless the government keeps the possibility of not respecting these formula.

Livret A : a passbook saving account cont'd

- 2004 : $\mathrm{LA}_t^{2004} := \frac{\overline{€3\mathrm{M}_{t-2}} + \frac{\mathrm{CPI}_{t-2}}{\mathrm{CPI}_{t-14}} - 1}{2} + 25 \text{ bps}$

- 2008 :

$$\mathrm{LA}_t^{2008} := \max\{\frac{\frac{\overline{€3\mathrm{M}_{t-2}} + \overline{\mathrm{Eonia}_{t-2}}}{2} + \frac{\mathrm{CPI}_{t-2}}{\mathrm{CPI}_{t-14}} - 1}{2} \,;\, \frac{\mathrm{CPI}_{t-2}}{\mathrm{CPI}_{t-14}} - 1 + 25 \text{ bps}\}$$

- 2009 :

$$\mathrm{LA}_t^{2009} := \min\left\{\mathrm{LA}_{t-6}^{2008} + 150 \text{ bps} \,;\, \mathrm{LA}_t^{2008}\right\} \quad \text{if } \mathrm{LA}_{t-6}^{2008} < \mathrm{LA}_t^{2008}$$

$$\mathrm{LA}_t^{2009} := \max\left\{\mathrm{LA}_{t-6}^{2008} - 150 \text{ bps} \,;\, \mathrm{LA}_t^{2008}\right\} \quad \text{if } \mathrm{LA}_{t-6}^{2008} > \mathrm{LA}_t^{2008}$$

Inflation Liquidity Gap

- The inflation liquidity gap is the function that assigns to each maturity the projected difference between the total liabilities remaining at this maturity and the total assets also remaining at this maturity which are sensitive to inflation movements.

We set

- T : Horizon of the scheduled gap
- The Livret A pays coupons at dates $T_1, \cdots, T_n$
- $D(t, T)$ represents the scheduled inflation liquidity gap at time T estimated at time t.

Projected Interest Rate Margin

- $D(t, T_i)\, R_{T_i}$ represents the investing liquidity gap income on interbank market (euribor)
- $D(t, T_i)\, \mathrm{LA}_{T_i}$ represents the amount due to the customer (i.e. the Livret A coupon)
- $\delta := T_{i+1} - T_i$

$$\mathrm{IRM}(t, T) := \mathbb{E}^{\mathbb{Q}}\Big[\sum_{i=1}^{n} e^{\int_t^{T_i+\delta} r_s\, ds} D(t, T_i) \Big(R_{T_i} - \mathrm{LA}_{T_i} \Big) \delta \Big]$$

Interest Rate Margin Hedging

We try to hedge IRM only by liquid products with m underlying X^k :

$$\begin{aligned}\mathrm{IRM}^{\mathrm{Hedge}}(t,T) &:= \mathbb{E}^{\mathbb{Q}}\Big[\sum_{i=1}^{n} e^{-\int_t^{T_i+\delta} r_s\,ds} D(t,T_i)\big(R_{T_i}-\mathrm{LA}_{T_i}\big)\delta\Big] \\ &+ \mathbb{E}^{\mathbb{Q}}\big[\sum_{k=1}^{m}\int_{-\infty}^{t}\phi^k(s,T)\sum_{\{j:t_j>t\}} e^{-\int_t^{T_j+\delta} r_u\,du}\big(\omega_s^k - X_{T_j}^k\big)\delta\,ds\big]\end{aligned}$$

$\phi^k(s,T)$ represents the nominal of hedging swap of maturity T contracted at time $s<t$.

Interest Rate Margin Hedging : cont'd (2)

Then we can find these nominals : $\forall k$

$$\frac{\partial \mathrm{IRM}^{\mathrm{Hedge}}(t,T)}{\partial X_t^k} = 0$$

To calculate the partial derivatives, we have now to choose an interest rate model to express $X_{T_j}^k = f(t, X_t^k)$.

Interest Rate Margin Hedging : cont'd (3)

For simplicity we can rewrite the IRM Hedge considering the following toy interest rate model :

- $m = 1$
- $X_t^1 := R_t$ with $R_{T_1} = R_t + \Delta$
- $n = 1$ and $t < T_1$ and $T = T_1$

$$\begin{aligned} \mathrm{IRM}^{\mathrm{Hedge}}(t, T_1) &:= P(t, T_1 + \delta) D(t, T_1) \big(R_{T_1} - \mathrm{LA}_{T_1}\big)\delta \\ &+ P(t, T_1 + \delta) \int_{-\infty}^{t} \phi^1(s, T_1) \big(\omega_s^1 - R_{T_1}\big)\delta \, ds \end{aligned}$$

Interest Rate Margin Hedging : cont'd (4)

Then we determine the nominal ϕ

$$\begin{aligned}\frac{\partial \mathrm{IRM}^{\mathrm{Hedge}}(t,T)}{\partial R_t} &= 0\\ P(t,T_1+\delta)D(t,T_1)\frac{\partial}{\partial R_t}(R_t+\Delta-\mathrm{LA}_{T_1})+P(t,T_1+\delta)\frac{\partial}{\partial R_t}\int_{-\infty}^{t}\phi^1(s,T_1)(\omega_s^1-R_t-\Delta)\,\delta\,ds &= 0\\ P(t,T_1+\delta)D(t,T_1)(1-\frac{\partial LA_{T_1}}{\partial R_t})-P(t,T_1+\delta)\phi^1(t,T_1) &= 0\end{aligned}$$

Then at each t we enter into a swap with nominal given by :

$$\phi(t,T_1)=D(t,T_1)(1-\frac{\partial \mathrm{LA}_{T_1}}{\partial R_t})$$

If this LA is a linear function of rates then we have $\phi(t,T_1)=D(t,T_1)(1-\alpha)$

Inflation : Regression Analysis

For simplicity we rewrite the Livret A formula as

$$\mathrm{LA_{T_i}}^{\mathrm{GRO}} := \max\{\frac{R_{T_i} + I_{T_i}}{2}\ ;\ I_{T_i} + 25 \text{ bps}\}$$

If we exprime I_{T_i} as a linear function of rate :

$$\hat{I_{T_i}} = \hat{\beta} R_{T_i} + \hat{\alpha}$$

We can rewrite :

$$\mathrm{LA_{T_i}}^{\mathrm{GRO}} := \max\{\frac{R_{T_i} + \hat{\beta} R_{T_i} + \hat{\alpha}}{2}\ ;\ \hat{\beta} R_{T_i} + \hat{\alpha} + 25 \text{ bps}\}$$

Inflation : Regression Analysis cont'd (2)

Then we have

$$\mathrm{IRM}^{\mathrm{Hedge}}(t, T) = f\left(R_{T_1}, \cdots, R_{T_n}\right)$$

If we choose $X_t^k = R_t$, then we can determine the different swaps with nominal $\phi(t, T)$

Spurious Regression

But the preceeding regression $\hat{I_{T_i}} = \hat{\beta} R_{T_i} + \hat{\alpha}$ can be spurious.

- If R_t and I_t are integrated of different order $\Longrightarrow$ the regression cannot be spurious
- If R_t and I_t are integrated of the same order and the test of cointegration is not rejected $\Longrightarrow$ the regression cannot be spurious

- X_t is first-order integrated if ΔX_t is an stationnary process, $\mathbb{E}(X_t) = \mu, \mathbb{V}(X_t) = \sigma^2, \mathbb{C}ov(X_t, R_{t+h}) = \gamma(h)$
- X_t and Y_t are cointegrated of order 1 if X_t and Y_t are cointegrated of order one and $\exists \gamma : X_t - \gamma Y_t$ is a stationnary process

Stationnarity test

- R_t is first-order integrated
- I_t is integrated of a dubious order 0 or 1 :

- Augmented Dickey–Fuller test reject the non-stationnarity of inflation
- Phillips–Perron test fails to reject the non-stationnarity of inflation

Stationnarity test - cont'd (2)

In many models inflation rate (ex Jarrow Yildirim) is considered as exchange rate between nominal and real worlds. And the exchange rate is often modelized as an ARFIMA process (fractionnaly integrated process) :

$$P(L)(1-L)^d I_t = Q(L)\epsilon_t$$

P, Q are polynomial function and L the lag operator

We fit this ARFIMA and we find the fractionnary integration order d = 0.49. If $d \in (-0.5, 0.5)$ is a stationary process.

Then stationarity is critical for inflation.

Regression Quality

We try to relate inflation to a linear function of rates

$$\hat{I}_t = \hat{\beta} R_t + \hat{\alpha}$$

but we fail to raise doubts about the quality of the regression.

Last but not least the interest rate explained only 22% of inflation (R-squared in regression model).

LA : Regression Analysis

We recall our problem : try to hedge IRM only by liquid products with m underlying X^k :

$$\mathrm{IRM}^{\mathrm{Hedge}}(t,T) := \mathbb{E}^{\mathbb{Q}}\Big[\sum_{i=1}^{n} e^{-\int_t^{T_i+\delta} r_s\,ds} D(t,T_i)\big(R_{T_i} - \mathrm{LA}_{T_i}\big)\delta\Big] + \mathbb{E}^{\mathbb{Q}}\Big[\sum_{k=1}^{m}\int_{-\infty}^{t} \phi^k(s,T) \sum_{\{j:t_j>t\}} e^{-\int_t^{T_j+\delta} r_u\,du}\big(\omega_s^k - X_{T_j}^k\big)\delta\,ds\Big]$$

Instead of relating the inflation inner the formula of LA, we try to explain directly the LA_t by two factors :

$$\mathrm{LA}_t = \alpha + \beta \mathrm{Eonia}_t + \gamma\Big(\mathrm{CMS}_t^{10Y} - \mathrm{CMS}_t^{2Y}\Big) + \epsilon_t$$

LA : Regression Analysis - cont'd (2)

The different statistics tests

- Augmented Dickey–Fuller
- Phillips–Perron
- KPSS

shows that the market rates and the client rate are integrated of the same order 1 and the series are cointegrated.

The regression cannot be spurious

LA : Regression Analysis cont'd (3)

Then we can inject the regression in the formula

$$
\begin{aligned}
\mathrm{IRM}^{\mathrm{Hedge}}(t, T_1) \quad := \quad & \mathbb{E}^{\mathbb{Q}}\big[e^{-\int_t^{T_2} r_s\, ds}\phi(t, T1)\big(R_{T_1} - \alpha + \beta \mathrm{Eonia}_t + \gamma\Big(\mathrm{CMS}_t^{10Y} - \mathrm{CMS}^{2Y}\Big)\big)\delta\big] \\
+ \quad & \mathbb{E}^{\mathbb{Q}}\big[\int_{-\infty}^{t} \phi^1(s, T)e^{-\int_t^{T_j} r_u\, du}\big(\omega_s^1 - R_{T_j}\big)\,\delta\, ds\big] \\
+ \quad & \mathbb{E}^{\mathbb{Q}}\big[\int_{-\infty}^{t} \phi^2(s, T)e^{-\int_t^{T_j} r_u\, du}\big(\omega_s^2 - \mathrm{Eonia}_{T_j}\big)\,\delta\, ds\big] \\
+ \quad & \mathbb{E}^{\mathbb{Q}}\big[\int_{-\infty}^{t} \phi^3(s, T)e^{-\int_t^{T_j} r_u\, du}\big(\omega_s^3 - \mathrm{CMS}_{T_j}^{2Y} + \mathrm{CMS}_{T_j}^{10Y}\big)\,\delta\, ds\big]
\end{aligned}
$$

Introduction

In this section we perform a risk analysis of the Livret A. We analyze the product as an exotic inflation indexed derivative product. In particular we focus on the sensitivities of the product to vanilla market instruments.

A three-factor HJM framework

It assumes that the CPI is an exchange rate between the nominal and real yields. The Jarrow-Yildirim model provides the non-arbitrage conditions between the three components : the nominal, the real and the inflation rate and leads to the following dynamics :

$$\begin{aligned} \frac{d\mathrm{CPI}_t}{\mathrm{CPI}_t} &= (r_t^n - r_t^r)dt + \sigma_{t,T}^{\mathrm{CPI}} dW^{\mathrm{CPI}_t} \\ dr_t^n &= \lambda^n(\theta_t^n - r_t^n)dt + \sigma_{t,T}^n dW_t^n \\ dr_t^r &= \lambda^r(\theta_t^r - r_t^r)dt + \sigma_{t,T}^r dW_t^r \end{aligned}$$

This three-factor model is calibrated to the term structure volatilities of three months caplets and 1Y CPI ratio options. Market Data of 31-03-2011 is used.

Livret A Swap

At every payment date T_{i+1} the product pays $N\mathrm{LA}_{T_i} * \delta$ where the rate LA_{T_i} is the Livret A fixed at date T_i.

One can rewrite the LA formula :

$$\begin{aligned} \mathrm{LA}_t^{\mathrm{GRO}} &:= \max\{\frac{r_t^n + \frac{\mathrm{CPI}_t}{\mathrm{CPI}_{t-12}} - 1}{2} \,;\, \frac{\mathrm{CPI}_t}{\mathrm{CPI}_{t-12}} - 1 + 25 \text{ bps}\} \\ &= I_t + 25 \text{ bps} + \max\{\frac{r_t^n - I_t}{2} - 25 \text{ bps} \,;\, 0\} \\ &= I_t + 25 \text{ bps} + \max\{\frac{r_t^r}{2} - 25 \text{ bps} \,;\, 0\} \end{aligned}$$

The last equation shows that the LA swap is similar to an inflation swap plus a cap on the real rate.

Sensitivity analysis

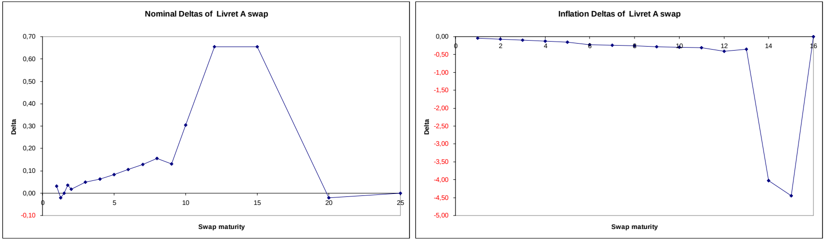


Figure: Deltas of Livret A swap.

That shows the sensitivities are naturally concentrated around maturity for both the nominal swaps and the inflation swaps.

Sensitivity analysis - cond't (2)

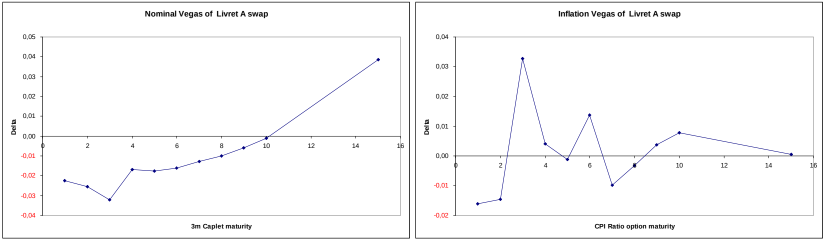


Figure: Vegas of Livret A swap.

Both inflation and nominal vegas are negligible.

Accumulator Livret A

The main drawback of the preceeding hedge product is that the Livret A swap does not truly hedges the Livret A passbook saving account. In practice interests are not actually perceived as a coupon by the client but capitalized in the saving account until the client decides to close its account.

The accumulator Livret A is a contract that gives a terminal payoff :

$$N\left(1 + \sum_{k:T_k<T}^{n} \prod_{i=k}^{n} \mathrm{LA}_{T_i}\right)$$

Sensitivity analysis

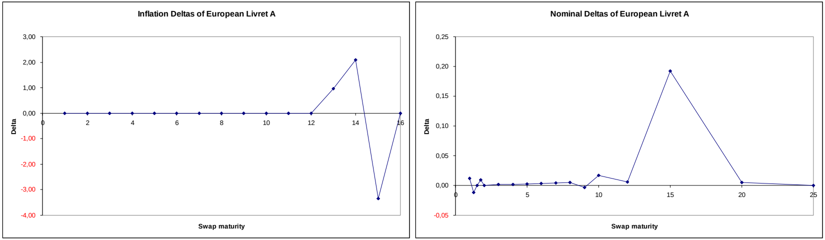


Figure: Deltas of European Livret A.

As for the LAS, that shows the sensitivities is naturally concentrated around maturity for both the nominal swaps and the inflation swaps.

Sensitivity analysis - cond't (2)

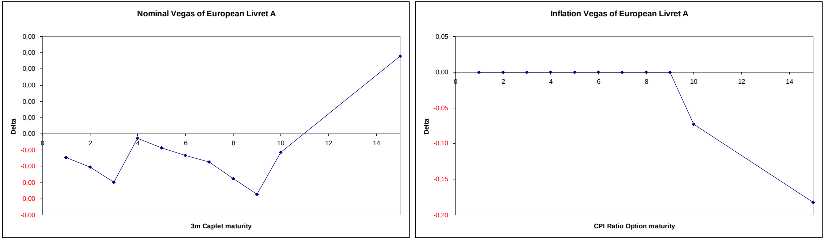


Figure: Vegas of European Livret A.

Both inflation and nominal vegas are more important than in the case of the Livret A Swap, and in order to properly hedge we should take this risk volatility risk into account.

Cancellable Accumulator Livret A

In fact, the client has the right to close his passbook account, obtaining its payoff at any date before maturity.

The cancellable accumulator Livret A is a contract that gives the right to exercice every 6 month, receiviving the capitalized interests on the notional :

$$N\left(1+\sum_{k:\,T_k<T^{ex}}^{n}\prod_{i=k}^{n^{ex}}\mathrm{LA}_{T_i}\right)\qquad T_{n^{ex}}=T^{ex}$$

Sensitivity analysis ALA

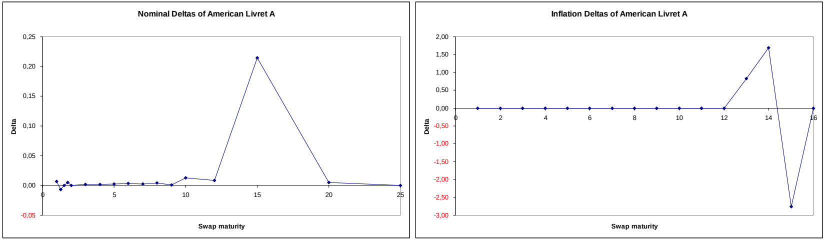
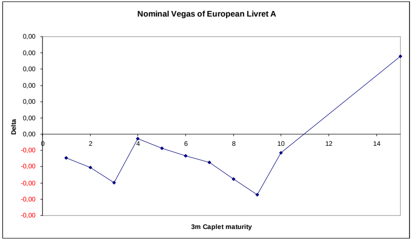


Figure: Deltas of American Livret A.

As for the LAS, that shows the sensitivities are naturally concentrated around maturity for both the nominal swaps and the inflation swaps.

Sensitivity analysis ALA cond't (2)



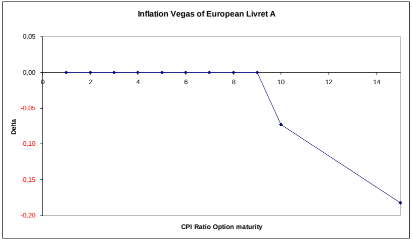


Figure: Vegas of American Livret A.

Inflation Vegas are almost ten times larger than in the case of the European Livret A. The inflation volatility risk thus becomes an important one for the American structure. This volatility risk mainly comes from the cancel option.

Comments

The ALA is the product which is actually sold by the bank to its clients.

In practice an "hedging horizon" is arbitrarily chosen for gap risk management.

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