WestLB

EXPERIENCES ON VALIDATION OF PD RATING SYSTEMS

Dr. Claudia Gregor-Lawrenz
Patrick Haverkate

Eurobanking 2011
Vienna – May 23, 2011

Table of contents

Table of contents

Introduction

WestLB uses the Internal Rating Based Approach

Credit risks of customers / transactions / project finances are estimated with own rating systems

Regulatory capital requirement is calculated based on the results of the internal rating systems

Introduction

Solvability regulation – Section 147

(1) The institution shall have robust systems in place to validate the accuracy and consistency of rating systems and processes and to estimate all relevant risk parameters. The institution shall demonstrate to BaFin that the internal validation process enables it to assess the performance of internal rating and risk estimation systems consistently and meaningfully.

(2) The institution shall regularly compare realised default rates with estimated PDs for each grade and, where realised default rates are outside the expected range for that grade, it shall specifically analyse the reasons for the deviation. ... Such comparisons shall make use of historical data that cover as long a period as possible. The institution shall document the methods and data used in such comparisons. This analysis and documentation shall be updated at least annually.

Introduction

Validation analyses used by WestLB

- Check of representativeness of validation data
- Descriptive analyses (portfolio overview, distribution of quantitative risk factors)
- Discriminatory power analyses (for end-ratings, sub-sections and risk factors, if reasonable)
- Benchmarking with external ratings (correlation, deviation and probability of default analyses)
- Special analyses of defaulted customers
- Backtesting (for rating years, rating classes and on overall rating history)
- And other

Introduction

Statistical tests used for validation analyses

- Binomial-test
- T-test
- Wilcoxon-test
- Kolmogorov-Smirnov-test
- (Extended) traffic light approach by Blochwitz
- Long-run test
- And other

Table of contents

Analysis of long-run PD-level

Solvability regulation – Section 130

(1) For IRBA exposures in the exposure classes Central governments, Institutions and Corporates and for PD-based IRBA-equity exposures, the institution shall estimate the PD per obligor grade for obligors from the **long-run average** of realised one-year default rates … .

(2) For purchased receivables not assignable as CCR exposures to the IRBA exposure class Retail claims, the institution may estimate ELs by obligor grade from **long run averages** of realised one-year default rates.

Analysis of long-run PD-level

Example

Default Year	Portfolio	Defaults	Default Rate	PD Forecast	p-value
2004	251	2	0,80%	0,22%	0,1063
2005	219	1	0,46%	0,34%	0,5227
2006	205	0	0,00%	0,31%	1
2007	191	0	0,00%	0,27%	1
2008	228	1	0,44%	0,33%	0,5294
2009	252	2	0,79%	0,29%	0,1664
Total	1346	6	0,45%	0,29%	0,2983

Binomial test for the number of customers, defaults and the overall average PD-level is not an adequate test to analyse the long-run behaviour of the rating system. It is important that the influence of one default year is independent from the corresponding number of customers.

→ application of a statistical test to analyse the overall PD-level

- traffic light approach by Blochwitz
- long-run test

Analysis of long-run PD-level

Traffic light approach by Blochwitz for 1 rating year

$d_{i,.}$ = number of defaults within rating year i,
$n_{i,.}$ = number of customers within rating year i,
p_i = average probability of default for rating year i

If $\frac{d_{i,.}}{n_{i,.}} < p_i$ => Green (default rate is lower than the probability of default.)

If $p_i < \frac{d_{i,.}}{n_{i,.}} < p_i + 0.842 \cdot \sqrt{\frac{p_i(1-p_i)}{n_i}}$ => Yellow (default rate is slightly higher than the PD.)

If $p_i + 0.842 \cdot \sqrt{\frac{p_i(1-p_i)}{n_i}} < \frac{d_{i,.}}{n_{i,.}} < p_i + 1.645 \cdot \sqrt{\frac{p_i(1-p_i)}{n_i}}$ => Orange (default rate is explicitly higher than the PD.)

If $p_i + 1.645 \cdot \sqrt{\frac{p_i(1-p_i)}{n_i}} < \frac{d_{i,.}}{n_{i,.}}$ => Red (default rate is much higher than the PD.)

(Note: The values 0.842 and 1.645 result from $\Phi^{-1}(0.8)$ and $\Phi^{-1}(0.95)$, respectively.)

WestLB

Analysis of long-run PD-level

Traffic light approach by Blochwitz for 4 rating years

- Conduction of the traffic lights approach by Blochwitz for 1 rating year to get a result for 4 consecutive rating years.
- The number of green, yellow, orange and red results from the individual rating years sum up to a final colour.
- The chronological order is not important.

Number of green results	0	0	0	0	0	0	0	1	0	0	1	0	0	1	0	1	0	1	1	0	2	1	1	2	0	1	2	2	1	2	3	2	3	3	4
Number of yellow results	0	0	0	1	0	1	0	0	1	2	0	1	2	0	2	1	3	0	1	3	0	1	2	0	4	2	0	1	3	1	0	2	0	1	0
Number of orange results	0	1	2	0	3	1	4	0	2	0	1	3	1	2	2	0	0	3	1	1	0	2	0	1	0	1	2	0	0	1	0	0	1	0	0
Number of red results	4	3	2	3	1	2	0	3	1	2	2	0	1	1	0	2	1	0	1	0	2	0	1	1	0	0	0	1	0	0	1	0	0	0	0
Total																																			

Analysis of long-run PD-level

Advantages of the traffic light approach by Blochwitz

- Comprehensive results – easy to interpret
- Application possible for 1 rating year and for 4 rating years
- Easy to implement
- Short calculation time
- Method is based on an equal weighting of rating years

Analysis of long-run PD-level

Disadvantages of the traffic light approach by Blochwitz

- Not applicable for 2, 3 or more than 4 rating years
  → in many cases part of the history is not captured in the final result

- The summing up of the results for single rating years to the joint result for 4 rating years is not comprehensible.
  → different approaches to derive the allocation of the year-results to an overall result led to no sensible results

- Chronological order has no influence

- No differentiation within one colour result

- Test results are hardly based on statistical consideration

Analysis of long-run PD-level

Long-run test

$d_{i,.}$ = number of defaults within rating year i,
$n_{i,.}$ = number of customers within rating year i,
p_i = average probability of default for rating year I
I = number of rating years

Test statistic $$\sum_{i=1}^{I}\left(\frac{d_{i,.}}{n_{i,.}} - p_i\right)$$

Analysis of long-run PD-level

Advantages of the long-run test

- Single result
- Exact statistical distribution used to calculate the p-value
- Application is possible for the entire rating history or the desired number of years
- Easy to implement
- Decision for recalibration can be made on the basis of one p-value.
- Method is based on an equal weighting of rating years.

Analysis of long-run PD-level

Disadvantages of the long-run test

- No approximate distribution of the test statistic
- Time consuming to conduct the test
- Chronological order is not important

Table of contents

Analysis of the spread of the rating system

Advance information

WestLB has a fixed master scale

➡ probability of default for a given rating class is constant for all rating systems and for all economic cycles

$$PD(A1) = p_1$$

$$\vdots$$

$$PD(D5) = p_{20}$$

Analysis of the spread of the rating system

Purpose

In addition to the calibration of the central tendency of a rating system the spread should be analysed.

Basis

Method of Marco J. van der Burgt* for low default portfolios uses discriminatory power analyses. Starting point is the cumulative accuracy profile (CAP). This method assigns a probability of default to all rating classes.

Notations

x_i = observed cumulative number of customers within rating class i

y_i = observed cumulative number of defaults within rating class i

I = number of rating classes

DR = default rate for the whole portfolio $= \frac{\sum_{m=1}^{I} y_m}{\sum_{m=1}^{I} x_m}$

z_i = cumulative amount of customers within rating class R $= \frac{x_I + x_{I-1} + \ldots + x_{i+1} + 0.5 \cdot x_i}{\sum_{m=1}^{I} x_m}$

* Marco J. van der Burgt: "Calibrating low-default portfolios, using the cumulative accuracy profile", Journal of Risk Model Validation Volume 1 Number 4.

Analysis of the spread of the rating system

Proceeding

Generating the CAP-curve on the basis of the final rating results and modelling the CAP-curve by adjusting the following function

$$y(x) = \frac{1-e^{-kx}}{1-e^{-k}}$$

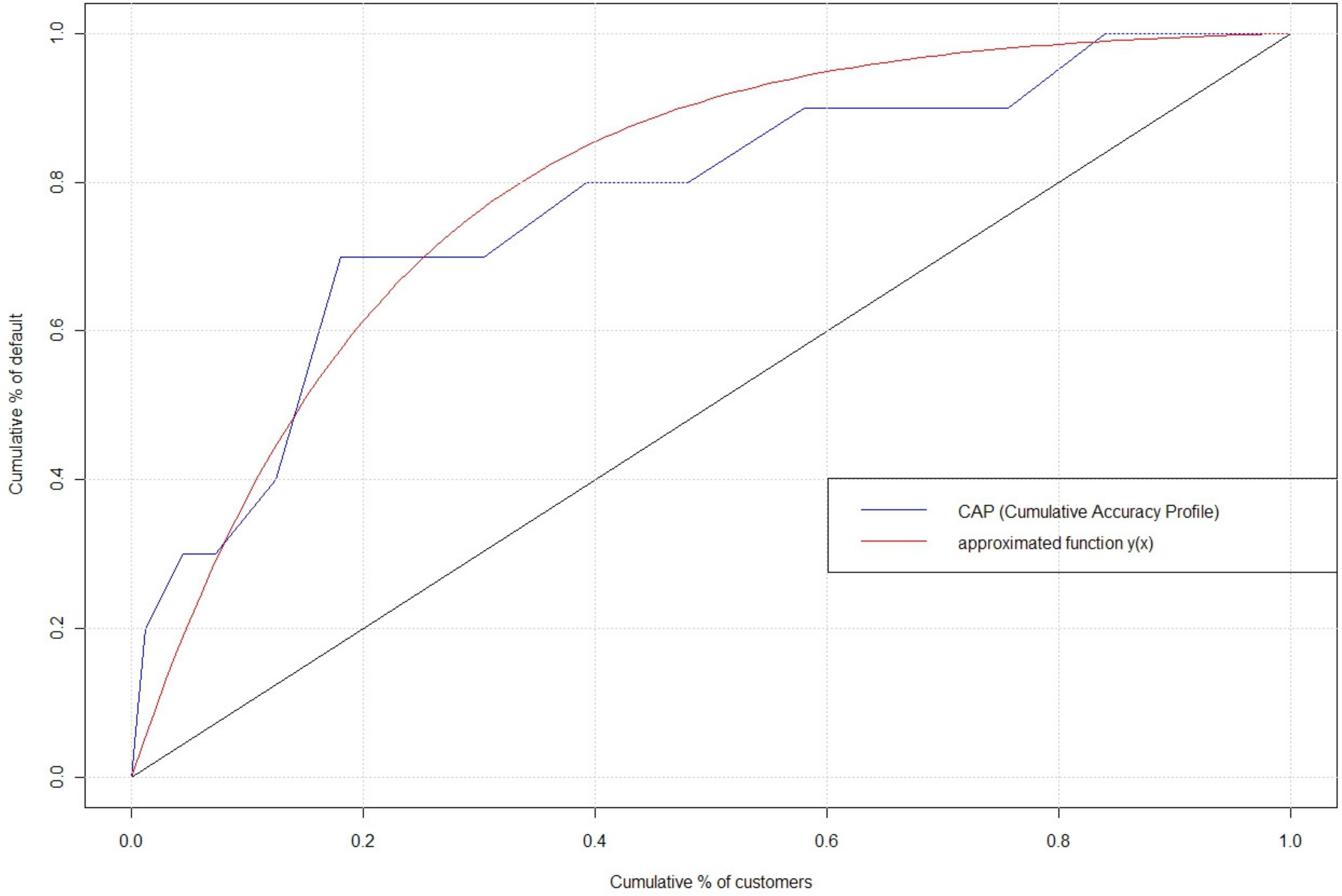
by adapting the parameter k ("concavity").

Minimising the following function E (e.g. by using Newton-Raphson) delivers the parameter k.

$$E = \sqrt{\frac{1}{I}\sum_{i=1}^{I}\left\{y_i - \frac{1-e^{-kx_i}}{1-e^{-k}}\right\}^2}$$

Note: The higher the value of k the better is the discriminatory power of the rating system. A negative value for k shows a "wrong sorting" of the customers.

Analysis of the spread of the rating system



Analysis of the spread of the rating system

Proceeding

Now it is possible to calculate the PDs for each rating class

$$PD(i) = \frac{k \cdot DR}{1 - e^{-k}} \exp(-k \cdot z_i)$$

Problem

The newly calculated PDs do not match with WestLB's master scale

Solution

Approximation of the newly calculated PDs by a logistic function, which models the relationship between the risk points belonging to the rating classes (mid points of the respective intervals) to the newly calculated PDs.

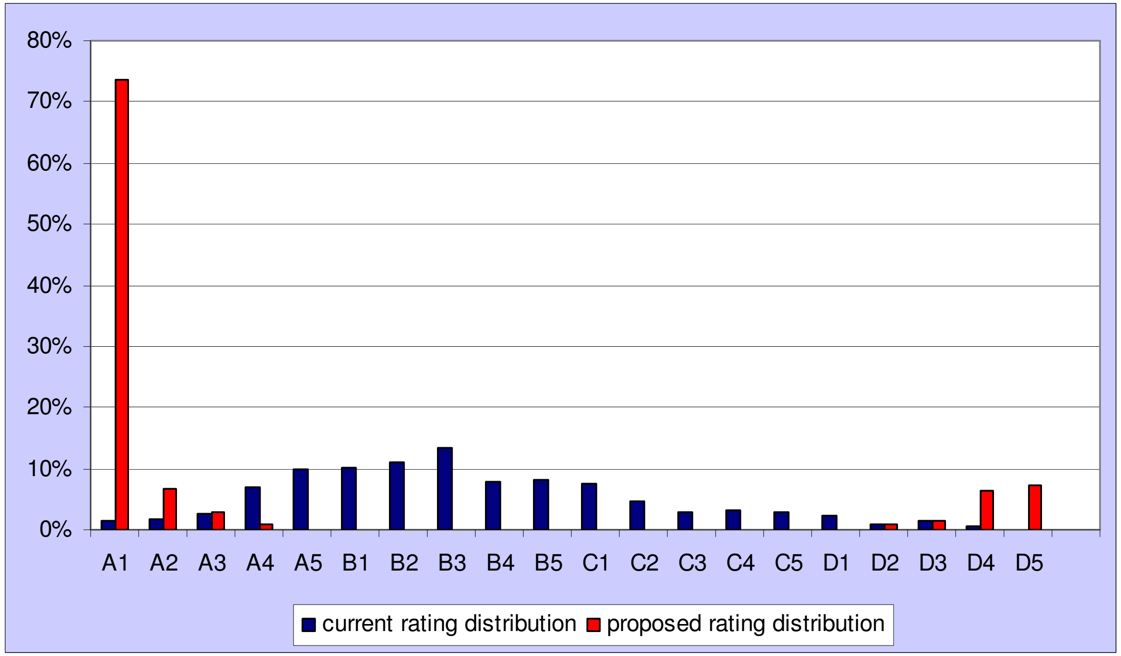
In a second step the central tendency can be adjusted by shifting the intercept parameter of the estimated logistic function.

Analysis of the spread of the rating system

Difficulties in practice

Method of van der Burgt is supposed to be used for low-default portfolios

➡ extreme values of the parameter k or the Accuracy Ratio are possible



<u>Approximately perfect discriminatory power</u>

For better rating classes a probability of default of ca. 0% is proposed.

The proposed probabilities of default for medium/poor rating classes increase strongly.

-> Rating distribution after transformation to master scale is concentrated within the best and worst rating classes.

Analysis of the spread of the rating system

Difficulties in practice

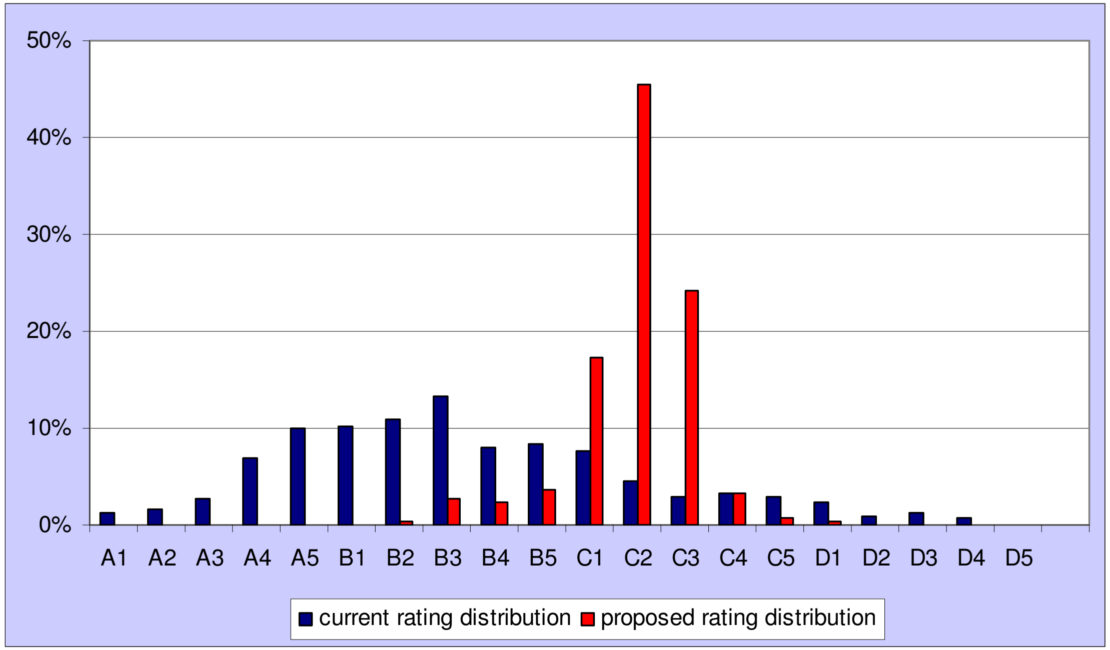
Method of van der Burgt is supposed to be used for low-default portfolios

→ extreme values for the parameter k or the Accuracy Ratio are possible

Very poor discriminatory power

Differences are only marginal between proposed probabilities of default for good rating classes and for poor rating classes.

-> Rating distribution after transformation to master scale is concentrated within few rating classes.



In several cases the presented method does not lead to economically meaningful results, especially the differentiation between customers is unsatisfactory.

Analysis of the spread of the rating system

Open issues

Method of van der Burgt leads to unsatisfactory results for many low-default-portfolios, but …

... a better method to analyse the spread of a rating system is unknown.

… a method to validate the spread as well as the central tendency of low-default-rating systems is unknown.

… a method to validate the spread as well as the central tendency of no-default-rating systems is unknown.

Analysis of the spread of the rating system

Current activities

WestLB is working on different methods to analyse the spread of a rating system.

- Results of one-sided binomial tests for rating classes can provide information about the quality of a rating system's spread. (Trend)
- Deviations of internal ratings to external ratings on the basis of rating classes can provide information about the quality of a rating system's spread. (Trend)
- Correlations of internal ratings to external ratings sub-divided by high, medium or low rating classes can provide information about the quality of a rating system's spread. (Trend)
- Execution of a logistic regression model with the default information.

Thank you for your attention